Negations of Quantifiers Lecture 10 Section 3.2

Robb T. Koether

Hampden-Sydney College

Thu, Jan 30, 2014

Robb T. Koether (Hampden-Sydney College)

Negations of Quantifiers

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# Negations of Quantified Statements

- 2 Negations of Conditionals
- 3 Necessary and Sufficient

# 4 Assignment

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- What would it take to make the statement "Everybody loves Raymond" false?
- What would it take to make the statement "Somebody loves Raymond" false?
- What would it take to make the statement "Nobody loves Raymond" false?

• The negation of the statement

$$\forall x \in S, P(x)$$

is the statement

$$\exists x \in S, \sim P(x).$$

- If  $\forall x \in \mathbb{R}, x^2 > 100$  is false, then  $\exists x \in \mathbb{R}, x^2 \le 100$  is true.
- What is the negation of the statement

$$\forall x \in \varnothing, x^2 > 100.$$

• The negation of the statement

$$\exists x \in S, P(x)$$

is the statement

$$\forall x \in S, \sim P(x).$$

- If " $\exists x \in \mathbb{R}, x^2 < 0$ " is false, then " $\forall x \in \mathbb{R}, x^2 \ge 0$ " is true.
- What is the negation of the statement

$$\exists x \in \emptyset, x^2 > 100.$$

- Let the domain be  $D = \{x_1, x_2, ..., x_n\}$ .
- The statement  $\forall x \in D, P(x)$  is equivalent to

$$P(x_1) \wedge P(x_2) \wedge \cdots \wedge P(x_n).$$

• By DeMorgan's Law, its negation is

$$\sim P(x_1) \lor \sim P(x_2) \lor \cdots \lor \sim P(x_n),$$

which is equivalent to  $\exists x \in D, \sim P(x)$ .

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- The statement  $\exists x \in D, P(x)$  is equivalent to

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• By DeMorgan's Law, its negation is

$$\sim P(x_1) \wedge \sim P(x_2) \wedge \cdots \wedge \sim P(x_n),$$

which is equivalent to  $\forall x \in D, \sim P(x)$ .

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- Are these statements equivalent?
  - "Any investment plan is not right for all investors."
  - "There is no investment plan that is right for all investors."

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- We should avoid using the word "any" when writing quantified statements.
- The meaning of "any" is ambiguous.
- "You can't put any person in that position and expect him to perform well."
- Instead, use "all," "some," "none," or other such words.

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## 2 Negations of Conditionals

3 Necessary and Sufficient

## Assignment

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• The negation of

$$\forall x \in S, P(x) \rightarrow Q(x)$$

is the statement

$$\exists x \in S, \sim (P(x) \rightarrow Q(x)),$$

which is equivalent to

$$\exists x \in S, P(x) \land \sim Q(x).$$

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- Consider the statement "All crows are black."
- Let *C*(*x*) be the predicate "*x* is a crow."
- Let *B*(*x*) be the predicate "*x* is black."
- The statement can be written formally as

$$\forall x, C(x) \to B(x)$$

or simply

$$C(x) \Rightarrow B(x).$$

What would constitute statistical evidence in support of this statement?

• The statement is logically equivalent to

$$\forall x, \sim B(x) \rightarrow \sim C(x)$$

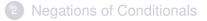
or simply

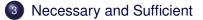
$$\sim B(x) \Rightarrow \sim C(x).$$

- What would constitute statistical evidence in support of this statement?
- Can you show that all crows are black without ever looking at a crow?

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1 Negations of Quantified Statements





## 4 Assignment

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• To say that *P* is necessary for *Q* means

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which is equivalent to

$$Q \rightarrow P$$
.

- Write the following statements as conditionals.
  - "To be a good man and a good citizen, it is necessary that you pay your taxes."
  - "To get a good job, you need a good education and know the right person."

• To say that *P* is sufficient for *Q* means

$$P \rightarrow Q$$
.

- Thus, "*P* is necessary for *Q*" and "*P* is sufficent for *Q*" are converses of each other.
- Write the following statements as conditionals.
  - "To be a good man and a good citizen, it is sufficient that you pay your taxes."
  - "Getting a good education or knowing the right person is sufficient to get a good job."

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• To say that *P* is necessary and sufficient for *Q* means

$$P \leftrightarrow Q.$$

• Write the following statements as conditionals.

- "For *n* to be a multiple of 6, it is necessary and sufficient that *n* be a multiple of 2 and a multiple of 3."
- "For *n* to be a multiple of 8, it is necessary, but not sufficient, that *n* be a multiple of 4."
- "For *n* to be a multiple of 4, it is sufficient, but not necessary, that *n* be a multiple of 8."

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#### Assignment

- Read Section 3.2, pages 108 115.
- Exercises 1, 3, 4, 9, 10, 17, 19, 20, 25, 37, 40, 43, page 115.

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